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Drag Reduction in Turbulent Flow Through Spanwise Wall Oscillations

Avijit Sharma[1], Kamal Yadav[2], Md. Ayaz J. Khan[3]
Tolani Maritime Institute, Pune

Turbulence exists across the whole human and natural environment, which is a matter of concern in terms of global energy consumption. Reducing the frictional force in turbulence to the greatest extent is becoming an urgent issue that needs to be resolved. Drag reduction is the science of flow improvement through the reduction of frictional pressure drop across a pipe or a channel. The paper examines the effectiveness of wall oscillation as a control scheme for drag reduction. Results include the effects of oscillation frequency and peak-to-peak amplitude of transverse wall oscillation on drag reduction at two different velocities lying in the low and high velocity zone. For the simulation, mesh is generated for the static conditions of the plate for the two inlet velocities which are theoretically validated for both two and three dimensional fluid domain. The simulation gives an insight on the phenomenon which occurs when the plate oscillates taking into consideration the analysis of computed local skin friction coefficient in the length-wise and span-wise directions over the plate as well as the total viscous drag value.

Keywords—Computational Fluid Dynamics (CFD), Drag, Fluent 14.0, K-epsilon, Mesh, Skin friction, Span wise, Turbulent flow, Hydrodynamics

1 INTRODUCTION

Appreciable progress has been made both in understanding of drag reduction mechanisms and in the development of techniques for obtaining drag reduction in the past ten years which has been further made possible by the advancement of computer capabilities and more or less imaginative control strategies.

Span-wise wall oscillation is an effective mechanism for achieving high drag reductions which has the imperative property of being open loop and thus contributes towards practical implementation without the need for complicated sensors to detect the flow state. In the early 1990s, Akhavan and co-workers proposed a simple active control technique. In this technique, oscillating the wall along the span-wise direction in a turbulent boundary layer flow leads to a sustained turbulent drag reduction.

The early studies of turbulent flow in a channel with wall oscillations using direct numerical simulation (DNS) was performed by Jung et al. DNS is a simulation in computational fluid dynamics in which the Navier–Stokes equations are numerically solved without any turbulence model. DNS studies performed for a turbulent channel flow showed that up to 40 % skin-friction drag reduction was possible at appropriate oscillation parameters. Their results were confirmed by the subsequent DNS. Experimental investigations conducted have confirmed and extended the DNS results. Past DNS studies have focused on channel flow with the benefits of periodic boundary conditions which reduce the computational toll. However, not much work has been done to explore the effects on a turbulent boundary layer using numerical simulations.

In this paper, the various state-of-the-art drag-reducing and energy saving technique are reviewed, primarily focusing on **span wise oscillations**.

S.NO	Drag Reduction Technologies	Drag Reduction Efficiency
1	Polymer additives drag reduction	More than 30%
2	Micro-Morphology drag reduction	8%-12%
3	Super-Hydrophobic drag reduction	More than 20%
4	Air bubble drag reduction	30%-35%
5	Heating wall drag reduction	20%
6	**Vibrating wall drag reduction**	**About 50%**
7	Composite drag reduction	About 50%

Table 1. Comparison of different drag reducing technologies

Various drag reducing technologies in turbulence based on boundary layer control. Also comparison of different drag reduction techniques is shown in Table1.

2 BOUNDARY LAYER

2.1 Boundary Layer Theory

When a fluid flows over a stationary surface, the fluid touching the surface is brought to rest by shear stress t_0 at the wall. The velocity increases from the wall to a maximum in the main stream of the flow. Ever since Stokes' has examined the motion of a fluid in contact with a harmonically vibrating plate, many authors have investigated the flow of a fluid between two oscillating parallel flat plates and through oscillating channels of various geometries.

As the boundary layer grows from zero when a fluid starts to flow over a solid surface and traverses over a greater length more fluid is decelerated by friction between the fluid layers close to the boundary. Hence thickness of slower layer increases. The fluid near the top of the boundary layer is dragging the fluid near the surface along. This dragging mechanism may be of one of the two types.

The first type occurs when the normal viscous forces (holds the fluid together) are large enough to exert drag effect on the slower moving fluid near the surface. If the boundary layer is thin then then the velocity gradient normal to surface, (du/dy) is large so by Newton's law of viscosity the shear stress (t = m.(du/dy) is also large. The analogous force is large enough to exert drag on the fluid close to the surface. This part of boundary layer is called laminar boundary layer.

As the boundary layer thickness becomes greater, velocity gradient becomes smaller and shear stress decreases until it is no longer adequate to drag the slow fluid near the surface. The viscous shear stresses have held the fluid particles in constant motion within the layers. They become insignificant as boundary layer thickness increases, eventually no longer able to hold the flow in layers and the flow starts to rotate and causes the fluid motion to become turbulent.

2.2 Skin Friction Drag

Skin friction drag is due to viscous shearing that takes place between the surface and the immediate layer of fluid. As fluid flows over a solid surface, the layer next to surface may become attached to it i.e. it wets the surface which is known as 'no slip condition'. The layers above the surface are moving, hence shearing occurs between them. The shear action between the wall and the first moving layer is called the wall shear. The skin drag is due to the wall shear stress τw and this acts on the wetted surface area. The drag force is hence: R = τw * wetted surface area. The dynamic pressure is the pressure resulting from conversion of kinetic energy of the stream into pressure and is defined by expression ϱu2/2.

The skin-friction coefficient is defined as

CD_f = (Drag Force)/(dynamic pressure*wetted area)

CD_f = 2R/(ϱU2*R/τw) = 2τw/ϱU2

Cf(x*) = 2/√Re ((du*)/dy) w; (from boundary-layer solution)

Therefore the skin-friction coefficient is determined from the velocity gradient at the wall (y=0). The skin-friction for all laminar boundary layers tends to approach zero with increasing Reynolds number.

2.3 Oscillating Wall Mechanism

The general mechanism by which oscillating walls achieve drag reduction is through suppression of bursts and sweeps. Specifically, the oscillating wall creates stroke boundary layer just above the viscous sub-layer which holds small localized vortices with spins opposing the oscillating movement.

The two most pronounced structural traits of near-wall turbulence are low and high speed "streaks," and quasi-stream-wise vortices, roughly aligned in the stream-wise direction. It is also recognized that there is a close interaction between the near-wall stream-wise vortices and the high skin-friction realized in turbulent boundary layers. The definite relationship, though, is still an active area of research.

The existence of these small vortices curbs the elongation and stretching of large vortices in stream wise direction. Since vorticity is measure of angular moment of rotating fluid, a reduction in mean vortex diameter directly translates into less vorticity. The oscillatory movement of wall also shifts the low speed streaks and quasi-stream-wise vortices closer to each other in stream-wise direction, thus disrupting the interaction between these structures. Burst and sweeps events that occur are changes in a dynamic turbulent flow that are essentially the result of a system attempting to achieve equilibrium. The intensity of the burst and sweeps are also reduced due to oscillation of the stroke layer which causes the vortices to pump high-rather than low speed fluid far off from the wall and splash low-rather than high speed fluid towards the wall, thereby reducing the stress in the near wall region of the boundary layer and thus reducing Drag.

Thus, if the formation of stream wise vortices is greatly suppressed, a boundary control scheme would be expected to lead to a greater reduction in the skin-friction drag.

Regarding practical applications, the oscillating wall technique may be still far from being practically applied. With the existing technological capabilities available, it is impractical to imagine large portions of the aircraft wing, fuselage surface, and ship's hull oscillating.

2.4 Effectiveness of Drag Reduction

Drag reduction depends on various factors which are as follows:

2.4.1 Frequency and Amplitude

The extent of drag reduction depends mainly on the frequency and amplitude of the wall oscillations. The wall oscillation is imposed through the boundary conditions for the span wise velocity component is

$$W_{wall} = W_a \sin(2\pi ft) \quad (1)$$

Where Wa is the velocity amplitude calculated from the channel flow rate.

The motion of wall surfaces directly correlates to the span wise displacement of the oscillating walls:

$$Z_{wall}=\Delta z/z \sin(2\pi ft) \quad (2)$$

Here, Δz is the peak-to-peak amplitude in the span wise direction.

Thus, there are two methods to perform the parametric study: the first method is to fix the velocity amplitude Wa and vary the frequency f, or vice versa; the second method is to fix the peak-to-peak amplitude Δz and change the frequency f, or vice versa.

In order to apply the second way multiplying 2πf on both sides of the equation (2)

$$W_{wall} = W_p \sin(2\pi ft) \quad (3)$$

Where, $W_{wall} = (Z_{wall})(2\pi f)$ and $W_p = (\Delta z / 2)(2\pi f)$, which is also termed the peak wall speed. Larger velocity amplitudes lead to a greater amount of drag reduction.

2.4.2 Peak Wall Speed

The extent of the drag reduction depends on both frequency and peak-to-peak amplitude. The amount of drag reduction due to a span-wise wall oscillation would better correspond to the peak wall speed Wp. Results from previous researchers initially the quantum of drag reduction increases rapidly with the peak wall speed and then the profile levels out.

2.4.3 Oscillation Orientation

The conceivable physical mechanism by which the wall oscillation results in a drag reduction involving the lateral shift of the stream wise vortices, which in turn disrupt the low-speed streak formation, an attempt is made to intensify the relative displacement of the near-wall streaks and the stream wise vortices above by dragging the channel wall obliquely with respect to the stream wise direction. The inclination angle γ is set to be 0°, 30°, 60° and 90°, with $\gamma = 0°$.

It is further observed that span wise wall oscillation ($\gamma = 90°$) has achieved the maximum drag reduction whereas the stream wise oscillation ($\gamma = 0°$) leads to the least gain. This finding is by Choi, H., Moin, P. and Kim, J., "Active Turbulence Control for Drag Reduction in Wall-Bounded Flows", in their active control study.

2.4.4 Reynolds Number

During the analysis of drag reduction, the Reynolds number is the only dynamic parameter required to be investigated in addition to the oscillation parameters themselves. Trujillo had conducted experiments to study the effect of Reynolds number on the drag reduction and the Reynolds number ranged from Re θ = 500, 950, 1400, to Re θ =2400, based on the momentum thickness. It was analyzed and concluded in his thesis that within the range of these Reynolds numbers the quantum of drag reduction that could be achieved was independent of the Reynolds number. Drag reduction is observed to become smaller as the Reynolds number is increased and beyond a certain threshold Reynolds number, the amount of drag reduction remains unchanged with subsequent increase in the Reynolds number.

2.4.5 Cost and Benefit

From a rational point of view, the cost of a drag reduction technique should be considered. In the present work using wall oscillations, the power saved is given as

$$\text{Power Saving} = (\tau\, w_0 - \tau\, w_{osci}).U \qquad (4)$$

Here, $\tau\, w_{osci}$ and $\tau\, w_0$, respectively, represent the stream-wise wall shear stress in the oscillating-wall channel and in the stationary-wall channel; U is the bulk velocity in the stream wise directions.

2.5 Law of the Wall

The law of the wall states that the average velocity of a turbulent flow at a certain point is proportional to the logarithm of the distance from that point to the "wall", or the boundary of the fluid region.

It is only technically applicable to parts of the flow that are close to the wall (<20% of the height of the flow), though it is a good approximation for the entire velocity profile of natural streams. Thus this law is used to calculate the distance of the first node point of the mesh based on the desired y^+ value to effectively capture the turbulent boundary layer.

3 CFD ANALYSIS

3.1 Static Turbulent Flow (Two Dimensional)

3.1.1 Problem Statement

To simulate and validate the turbulent flow the following problem statement is considered with reference to "Fluid Mechanics, third edition", by Yunus A.Cengel, John M. Cimbala.

A plastic boat whose bottom surface can be approximated as a 1.5m wide, 2m long flat surface is to move through water at 15 degree C at speeds up to 30km/hr. Determine

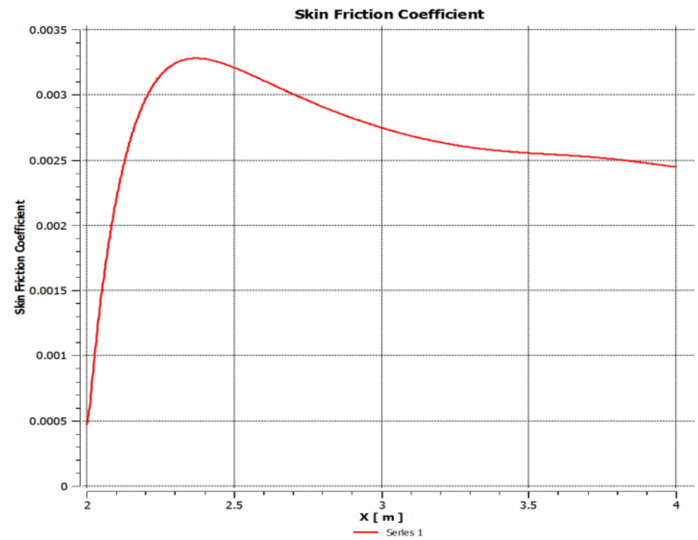

Fig 1. Local Skin Friction Coefficient along the Length of the Plate

the frictional drag exerted by the boat on the water?

Assumptions:

- The flow is steady and incompressible.
- The water is calm (no significant waves or currents).
- The water flow is turbulent over the entire surface of the plate.
- The bottom surface is flat and smooth.

3.1.2 Fluid Domain and Mesh Generation

The boundary layer height for a plate of length 2m with a flow velocity of 8.33m/s above it is 0.0303m. Thus the fluid domain height is taken as more than 10 times the boundary layer thickness i.e. 0.8m. Mapped face meshing is undertaken with the inlet and outlet divided into 100 divisions each. The plate is divided into 200 divisions so as to get a y^+ equal to 80.

3.1.3 Setup and solution

The global meshing controls include setting the relevance center to fine and smoothing to high, other parameters to the default based on the geometry. The standard

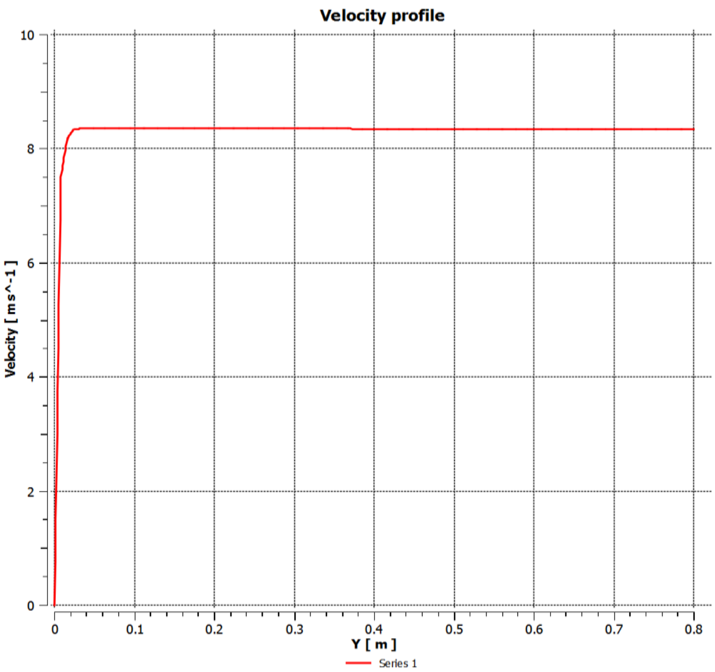

Fig 2. Velocity Profile in Y direction at 1.5m

K-epsilon model is selected with near wall treatment set to standard wall functions. Boundary conditions are set as per the requirement with inlet velocity x component at 8.33m/s and outlet pressure at 0 Pascal (gauge).

In the solution methods, momentum, turbulent kinetic energy and turbulent dissipation rate are set to second order upwind equations so as to increase the accuracy of the result. The calculations are done for 1000 iterations and it converges at 167th iteration.

3.1.4 CFD Post Results and Theoretical Validation

Viscous force of 187.889 N/m on the plate of length 2m is computed. The Reynolds number at the end of the bottom surface of the boat is as follows:

Re = ϱVd/μ = ((999.1 kg/m3).(30/3.6 m/s).(2m))/(1.138 x 10-3 kg/ms) = 1.463 x 107

The flow is assumed to be turbulent over the entire surface. Then the average friction coefficient and the drag force acting on the surface becomes

C_D = (0.074)/(Re^(1/5) = 0.074 / (1.463 x 107) ^ (1/5)
= 0.00273.

F_D = 1/2 ϱ v2 CD A

F_D = 1/2 (999.1 kg/m3)⟦(30/3.6 m/s)⟧^2 (0.00273) (1.5x2 m2) (1N/1kg.m/s2) = 284.1 N

Thus, drag per meter, 284.1/1.5 = 189.4N/m.

3.2 Static Turbulent Flow (Three Dimensional)

3.2.1 Fluid Domain and Mesh Generation

The geometry is constructed in the Design Modeler of Workbench 14.0. The domain size is 3m x 1m x 8m in the span wise direction, height in the y direction and length along the positive x direction respectively. Plate is placed at the centre of the domain at a distance of 2m from the inlet having a dimension of 0.5m wide, 2m in length and a thickness of 2mm.

In order to save on the computational power required, inflation is used around the six faces of the plate with inflation option set to First Layer Thickness having a height of 8mm and number of layers being three with a growth rate of one to effectively capture

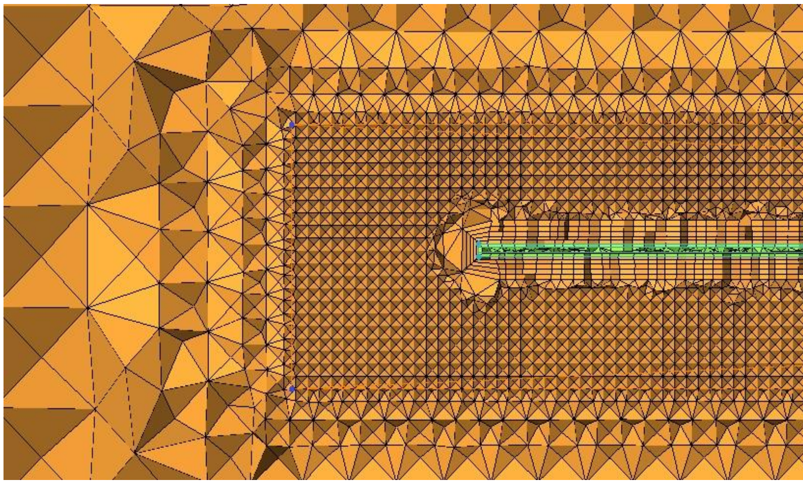

Fig 3. Mesh Inflation based on first layer thickness

the boundary layer. Mapped face Meshing is selected for the six plate faces so that the plate surface can be divided into the required number of quads so as to have uniform spacing and orientation of node points over the plate surface for better computational results. Edge sizing is chosen to divide all 8 edges of the surface into n number of edges with length of 1cm each.

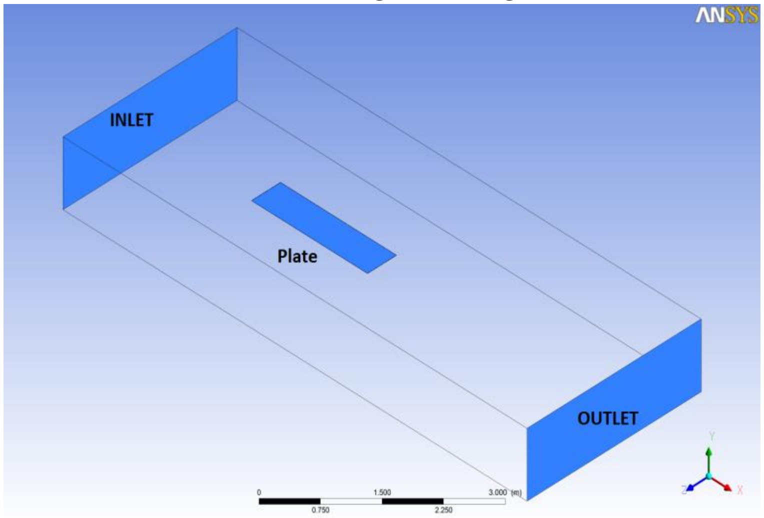

Fig 4. Fluid Domain

The total number of node points and elements generated are 1063560 and 5517789 respectively.

3.2.2 Setup and Solution

The standard K-epsilon model is selected with near wall

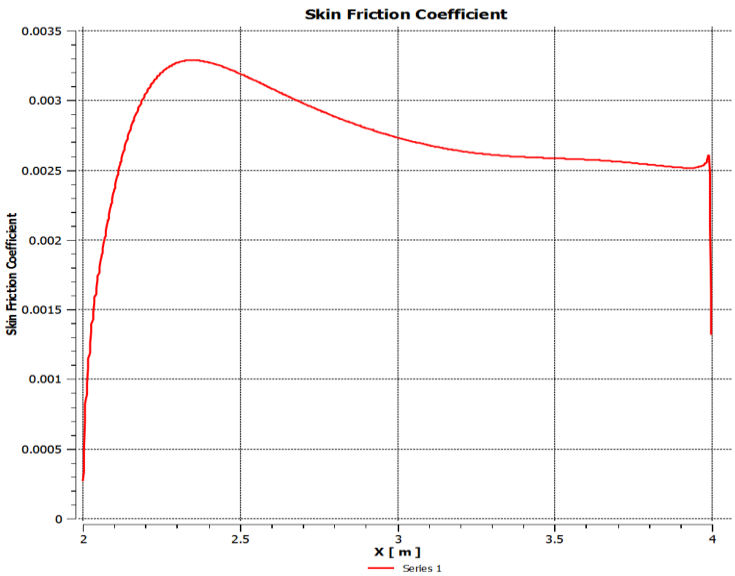

Fig.5. Local Skin Friction Coefficient along the length of the plate.

treatment set to standard wall functions. In the boundary condition settings the wall-solid i.e. the surface bounding the domain length wise is set to symmetry to simulate an open channel flow. Inlet velocity x component is set at 8.33m/s, with specification Method set to Intensity and Viscosity Ratio having Turbulent intensity(%) and Turbulent viscosity ratio equal to 10. The calculations are done for 1000 iterations and it converges at 394th iteration.

3.2.3 CFD Post results and Theoretical validation

Viscous force of 191.12282 N/m is obtained on the plate of length 2m with all six faces exposed to the fluid flow. Also the skin friction coefficient along the length of the plate is as depicted in the figures 4.

Length of plate = 2m, Breadth = 0.5m

Drag force per meter along the length of plate = 189.4 N/m

Therefore drag over the two surface of the plate = 2 X 0.5 X 189.4 = 189.4 N

Computational Values = 191.12 N

Thus the static turbulent flow is validated for three dimensional domain.

3.3 Transient Turbulent Flow (Three Dimensional)

3.3.1 Setup and Solution

Fluent 14.0 is opened through Visual studio 2013 x64 Cross Tool Command Prompt which acts as compiler for the UDF to be imported for the simulation of span wise oscillating plate.

The UDF is as follows for the first oscillation simulation performed.

```
 #include "udf.h"
#include "math.h"
#include "dynamesh_tools.h"
#define z 0.04
#define f 0.5

DEFINE_CG_MOTION(object, dt, vel, omega, time, dtime)

{
  /* reset velocities */
  NV_S(vel, =, 0.0);

  /* compute velocity formula */
    vel[2]=z*M_PI*f*sin(2*M_PI*f*time);

  printf("\n z_velocity = %g \n",vel[2]);}
```

Where z represents peak to peak amplitude of plate oscillation in the span wise direction, f represents frequency of oscillation, Vel[2] represents the equation of velocity in the span wise direction (Z direction in this case).

The dynamic mesh setting is selected with smoothing and remeshing options selected and plate is selected as dynamic mesh zone acting as a rigid body.

In the Run Calculation section following are the values set

- Time Step Size (s) = 0.02
- Number of Time Steps = 65
- Extrapolate variables is selected to as to get faster convergence.
- Max Iterations/Time Steps = 30

3.3.2 CFD Post results

Viscous force of 167.53603 N/m is obtained on the plate of length of 2m with all six faces exposed to the fluid flow, vibrating at a frequency of 0.5 Hz and peak to peak wall amplitude of 4cm.

The above value indicates a drag reduction of nearly 13 per cent.

Similar to the above simulation a number of simulations are performed for different data sets based on the theory review

For the case of 0.5m/s inlet velocity simulation, the solution for the static model converges at 493rd iteration. Simulations are done at different frequencies and peak to peak wall amplitude for inlet velocity of 0.5m/s which is depicted in Table 2.

The lowest drag value for this case is found out to be 1.1110 N, thus giving a drag reduction of nearly 11 per cent with respect to the static drag value.

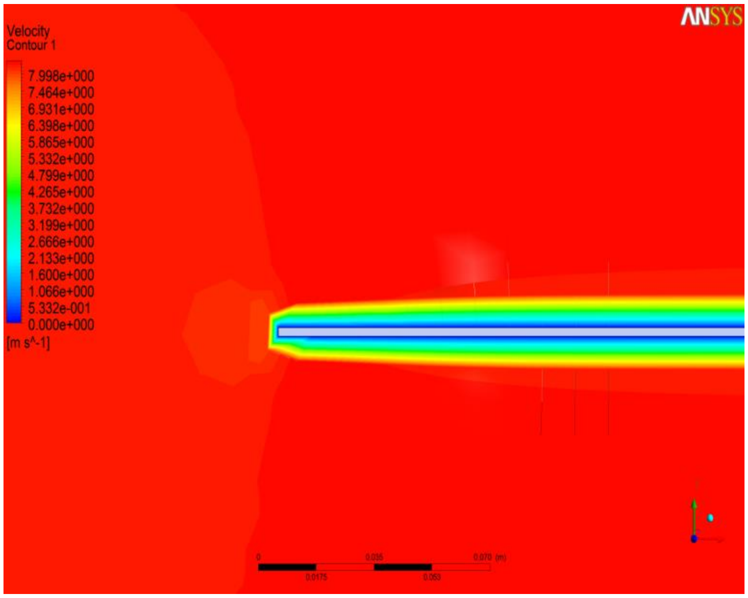

Fig 6. Velocity contour at leading Edge (Static)

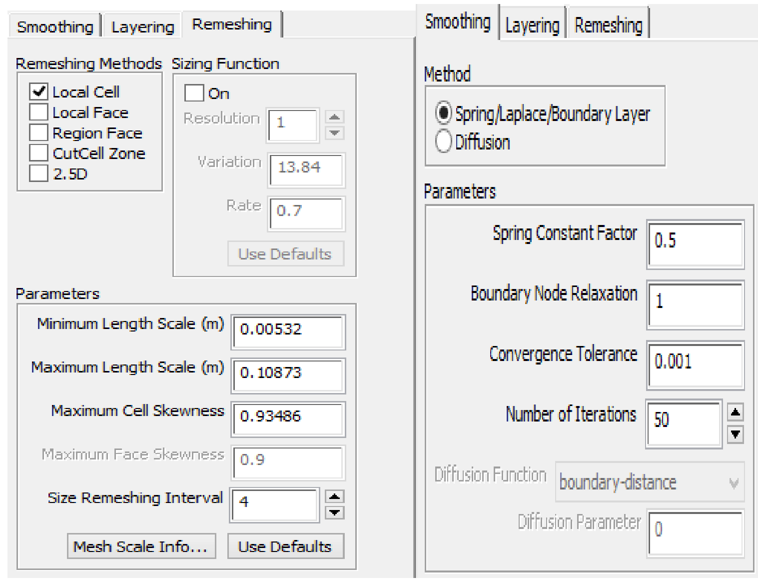

Fig 8. Smoothing and Remeshing Setting.

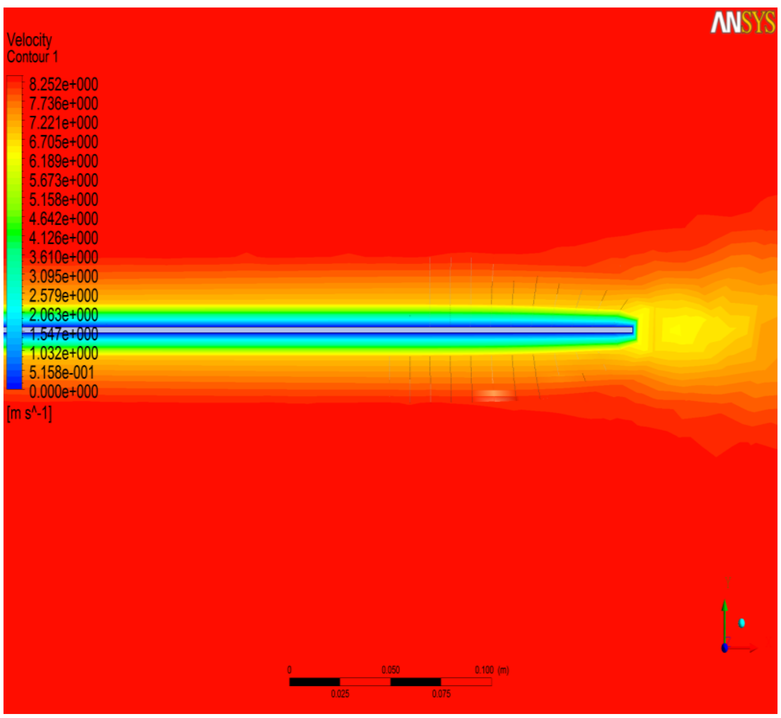

Fig 7. Velocity contour at Trailing Edge (Static)

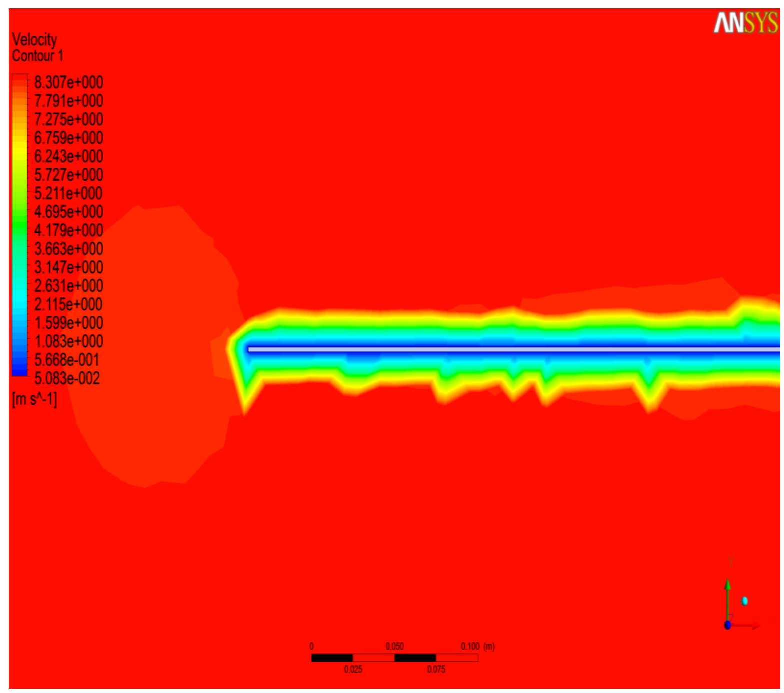

Fig 9. Transient Velocity Contour along the length of Oscillating Plate at leading Edge (f = 0.5Hz, Z = 4cm).

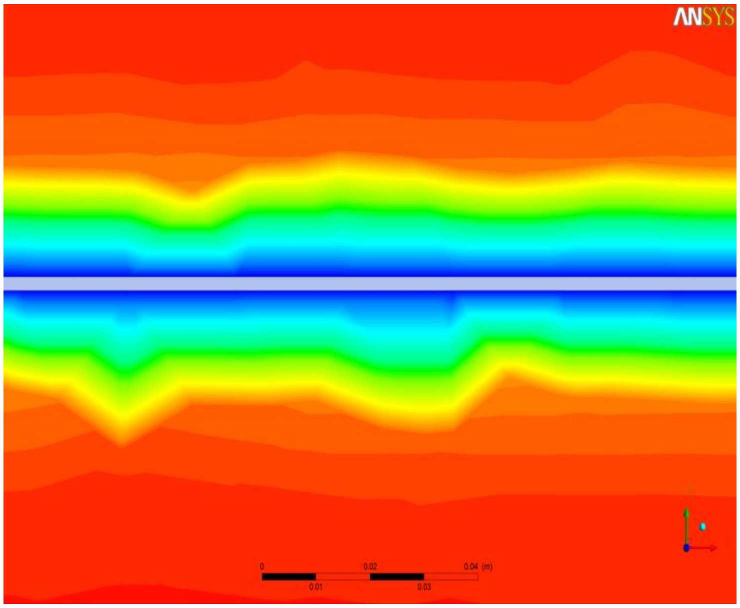

Fig 10. Magnified View of Velocity Contour along the Length of Oscillating Plate (f = 0.5Hz, Z = 4cm).

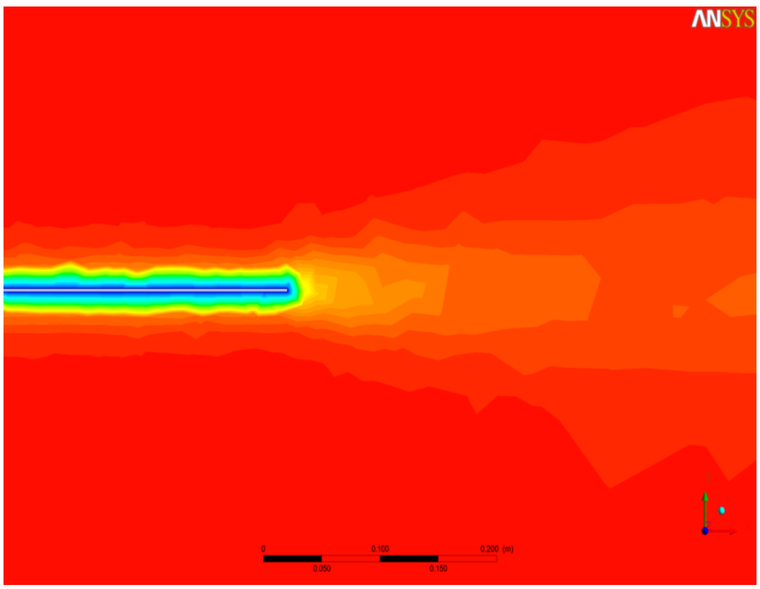

Fig 11. Transient Velocity Contour along the length of Oscillating Plate at trailing Edge (f = 0.5Hz, Z = 4cm).

4 Observation and Discussion

In this study, oscillation of a plate in a turbulent open channel flow subjected to a span wise wall oscillation has been simulated through computational fluid dynamics and the static model for each case is theoretically validated.

The following observations are made from the above analysis:

1. There is difference of 3-4 per cent between the theoretical and the computed drag value for the plate due to truncating errors in the CFD calculations.
2. For different values of frequency and span wise displacement for oscillation, different values of drag are achieved giving a maximum drag reduction for the data set as 13 per cent and 10 percent for flow velocities of 8.33m/s and 0.5m/s respectively.
3. It is observed that in case of higher velocity of 8.33m/s as well as lower velocity of 0.5m/s, the reduction of span wise displacement results in further lowering of the drag value at the same frequency. With lower velocity of 0.5m/s with increase in frequency of oscillation for the same value of displacement, there is an increase in the drag value.
4. In the static simulation, initially the local skin friction coefficient increases rapidly and reaches a peak value at approximately 0.5m along the length of the plate, after which it starts to decrease becoming steady at the end of the plate.
5. In case of span wise wall oscillation of the plate, the local skin friction coefficient increases slowly as compared to the static simulation and varies along the length of the plate without reaching a very high value as in the case of static simulation.

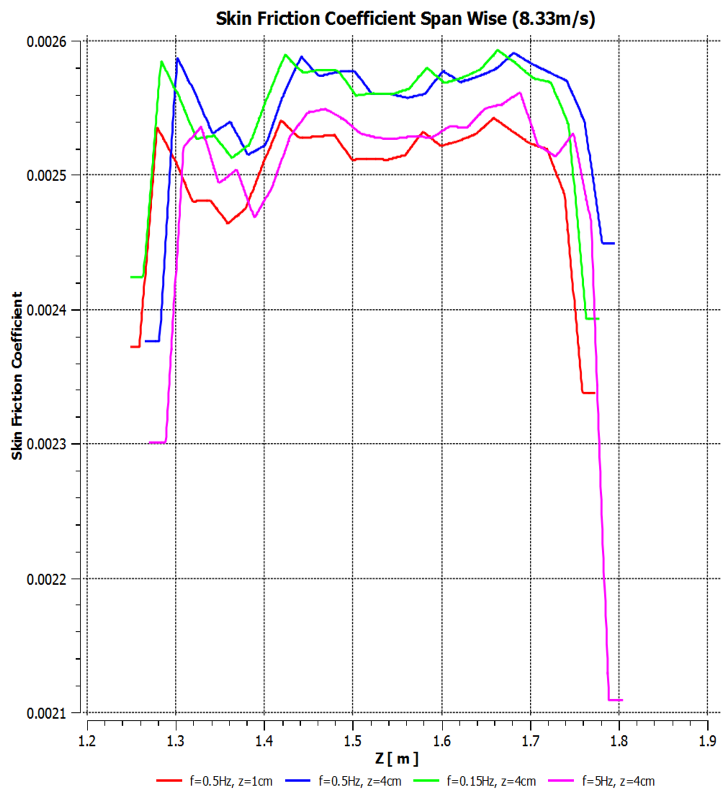

Fig 12. Local skin friction coefficients in the transverse direction at 1.5m length of plate for various frequencies and peak to peak displacement

5 Conclusion

There is general understanding that oscillations of a suitable frequency and amplitude can restrict turbulent activity therefore ensuring feasible skin friction drag reduction by

modifying the boundary layer conditions and initiating order in a previously chaotic turbulent flow. The primary independent variables for drag reduction via Span wise oscillating walls are therefore the maximum span wise displacement (Z), the maximum span wise wall velocity (Wa), and the period (T).
A riveting facet of research in this field is the study of how far past the oscillating plates effects prevail. This is of prime importance because it determines if a series of oscillating plates in sequence can be used to further promote energy savings.

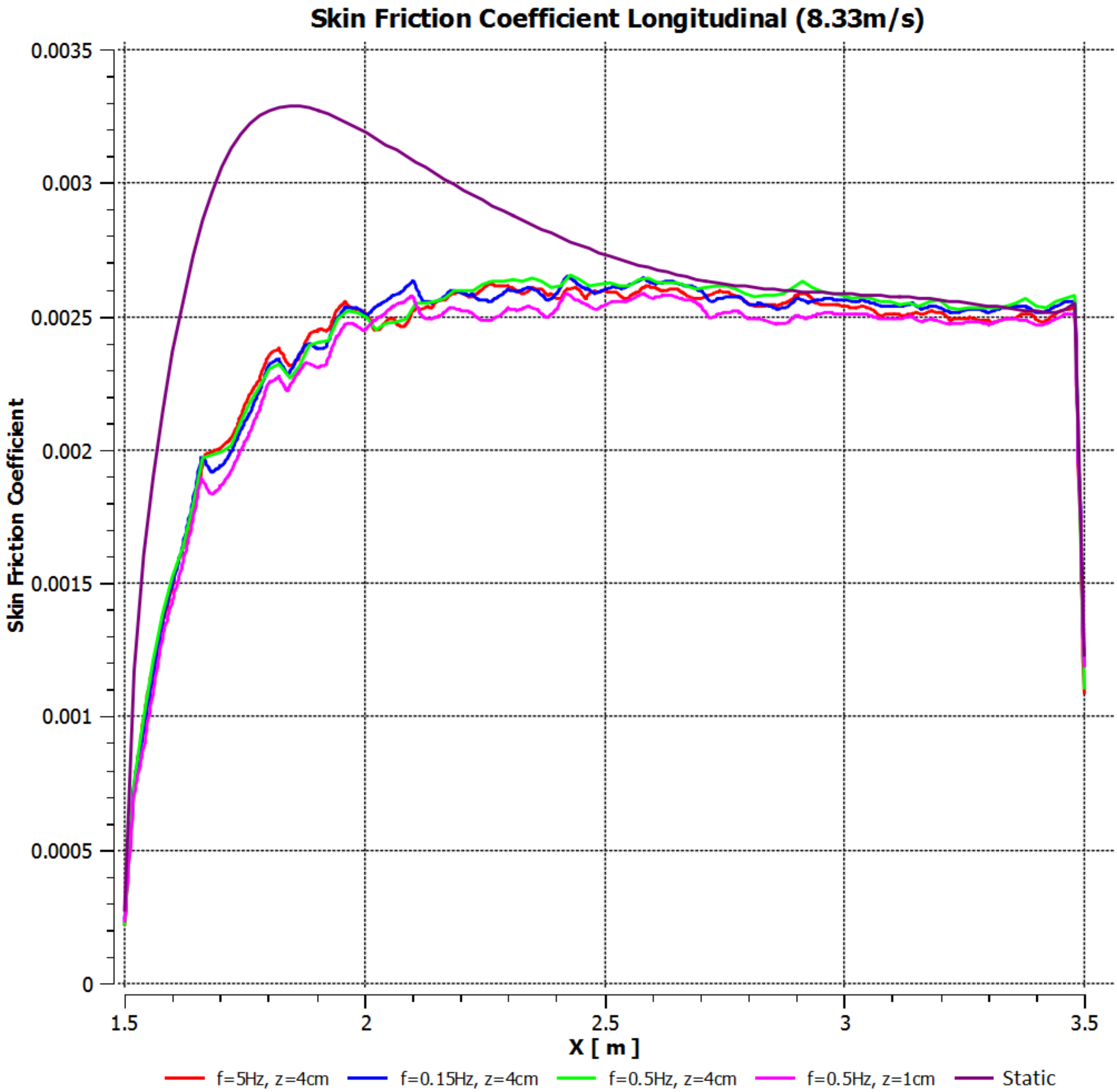

Fig 13. Local skin friction coefficients along the length of plate for various frequencies and peak to peak displacement.

TABLE 2
COMPUTED VISCOUS DRAG FOR THE DATA SET

Velocity of Water at Inlet (m/s)	Frequency (Hertz)	Displacement (Meter)	Viscous Drag (N)
8.333	0.5	0.01	163.85
	0.5	0.04	167.12
	5	0.04	166.53
	0.15	0.04	167.65
	0	0	191.122
0.5	5	0.04	1.3852
	0.5	0.04	1.1664
	0.15	0.04	1.1124
	1	0.005	1.1110
	0	0	1.241

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REFERENCES

[1] A. Gerasimov, "Modeling Turbulent Flows with FLUENT," Europe, ANSYS, lnc. 2006.

[2] Auteri, F., Baron, A., Belan, M., Campanardi, G. & Quadrio, M. 2010 Experimental assessment of drag reduction by traveling waves in a turbulent pipe flow. Phys. Fluids 22 (11), 115103/14.

[3] Baron, A. and Quadrio, M., "Turbulent Drag Reduced by Spanwise Wall Oscillations", Appl. Sci. Research, Vol. 55, (1996), 311-326.

[4] Bernard, P. S., Thomas, J. M. and Handler, R. A., "Vortex Dynamics and the Production of Reynolds Stress", J. of Fluid Mech., Vol. 253, (1993), 385-419W.-K.

[5] Berger, T. W., Kim, J., Lee, C. and Lim, J., "Turbulent Boundary Layer Control Utilizing the Lorentz Force", Phys. Fluids, Vol. 12, No. 3, (2000), 631-649.

[6] Brooke, J. W. and Hanratty, T. J., "Origin of Turbulence-Producing Eddies in a Channel Flow", Phys. Fluids A., Vol. 5, (1993), 1011-1022.

[7] Choi, J. I., Xu, C. X. and Sung, H. J., "Drag Reduction by Spanwise Wall Oscillation in Wall-Bounded Turbulent Flows", AIAA J., Vol. 40, No. 5, (2002), 842-850.

[8] Du, Y., Symeonidis, V. and Karniadakis, G. E., "Drag Reduction in Wall-bounded Turbulence via a Transverse Traveling Wave", J. Fluid Mech., Vol. 457, (2002), 1-34.

[9] Gad-el-Hak, M., "Flow Control: Passive, Active and Reactive Flow Management", Cambridge University Press, New York, U.S.A., (2000).

[10] Gouder, K. 2011 Turbulent Friction Drag Reduction Using Electroactive Polymer Surfaces. PhD thesis, Imperical College, London.

[11] Hermann Schlichting, Boundary Layer Theory [2000].

[12] John J. Bertin, Aerodynamics for Engineers, 4th edition, 2002.

[13] Karniadakis, G. E. and Choi, K. S., "Mechanisms on Transverse Motions in Turbulent Wall Flows", Annu. Rev. Fluid Mech., Vol. 35, (2003), 45-62.

[14] Kasagi, N., Hasegawa, Y. & Fukagata, K. 2009 Towards cost-effective control of wall turbulence for skin-friction drag reduction. Advances in Turbulence XII, vol. 132, pp. 189–200. Springer.

[15] Kim, J., "Control of Turbulence Boundary Layers", Phys. Fluids, Vol. 15, No. 5, (2003), 1093-1105.

[16] Laadhari, F., Skandaji, L. and Morel, R., "Turbulence Reduction in a Boundary Layer by a Local Spanwise Oscillating Surface", Phys. Fluids, Vol. 6, No. 10, (1994), 3218-3220.
[17] Lu, S. S. and Willmarth, W. W., "Measurements of the Structure of the Reynolds Stress in a Turbulent Boundary Layer", J. Fluid Mech., Vol. 60, (1973), 481- 511.
[18] Orlandi, P. and Jimenez, J., "On the Generation of Turbulent Wall Friction", Phys. Fluids, Vol. 6, No. 2, (1994), 634-641.
[19] Miyake, Y., Tsujimoto, K. and Takahashi, M., "On the Mechanism of Drag Reduction of Near-Wall Turbulence by Wall Oscillation", JSME Inter. J. Series B., Vol. 40, No. 4, (1997), 558-566.
[20] MOIN P., MAHESH K. Direct numerical simulation: a tool in turbulence research[J]. Annual Review of Fluid Mechanics, 1998, 30: 539-578.
[21] N. Patten, T. M. Young, and P. Griffin , Design and Characteristics of New Test Facility for Flat Plate Boundary Layer Research, World Academy of Science, Engineering and Technology 58, 2009.
[22] Quadrio, M. and Sibilla, S., "Numerical Simulation of Turbulent Flow in a Pipe Oscillating Around its Axis", J. Fluid Mech., Vol. 424, (2000), 217-241.
[23] R. Bashakaran, and C. Lance, "Introduction to CFD," New York, Cornell University.
[24] Robert H. Kraichnan, Pressure Fluctuations in Turbulent Flow over a Flat Plate, J. Acoust. Soc. Am. Volume 28, Issue 3, pp. 378-390 (May 1956).
[25] Schoppa, W. and Hussain, F., "Coherent Structure Dynamics in Near-Wall Turbulence", J. Fluid Dynamics Research, Vol. 34, (2000), 175-198.
[26] P.R. Spalart, S.R. Allmaras, one-equation turbulence model for aerodynamic flows, Technical Report AIAA-92-0439, American Institute of Aeronautics and Astronautics, 1992.
[27] SREENIVASAN K, R., ANTONI R. A. The phenomenology of small-scale turbulence[J]. Annual Review of Fluid Mechanics, 1997, 29: 435-472.
[28] Tardu S. F., Binder, G. and Blackwelder, R. F., "Turbulent Channel Flow with Large-Amplitude Velocity Oscillation", J. Fluid Mech., Vol. 267, (1994), 109-151.
[29] Touber, E. & Leschziner, M.A. 2012 Near-wall streak modification by spanwise oscillatory wall motion and drag reduction mechanisms. J. Fluid Mech. 693, 150–200.
[30] Trujillo, S. M., "An Investigation of the Effects of Spanwise Wall Oscillation on the Structure of a Turbulent Boundary Layer", Ph.D. Dissertation, The Univ. of Texas at Austin, (1999).
[31] Wu, S. L., "An Investigation of the Reynolds Number Effect on the Drag Reduction by Spanwise Wall Oscillation", Master Thesis, The Univ. of Texas at Austin, (2000).
[32] XU C. X., CHOI J. and SUNG H. J. Suboptimal control for drag reduction in turbulent pipe flow[J]. Fluid Dy- namic Research, 2002, 30(4): 217-231.
[33] Yunus A.Cengel, John M. Cimbala, FLUID MECHANICS, third edition.
[34] www.cfd-online.com.
[35] Zhao, H., Wu, J. Z. and Luo, J. S., "Turbulent Drag Reduction by Traveling Wave of Flexible Wall", J. Fluid Dynamics Research, Vol. 34, (2004), 175-198.